Contingency-Constrained Unit Commitment With Intervening Time for System Adjustments

Zhaomiao Guo, Richard Li-Yang Chen, Neng Fan, and Jean-Paul Watson, *Member, IEEE*

***Abstract*—The N-1-1 contingency reliability criterion considers the consecutive loss of two components in a power system, with intervening time for system adjustments between the two losses. In this paper, we consider the problem of optimizing generation unit while ensuring the N-1-1 criterion. Due to the coupling of time periods associated with consecutive component losses, the resulting problem yields a very large-scale mixed-integer linear optimization model. For efficient solution, we introduce a novel branch-and-cut algorithm using a temporally decomposed bilevel separation oracle. The model and algorithm are assessed using multiple IEEE test systems, and a comprehensive analysis is performed to compare system performance across different contingency criteria. Computational results demonstrate the value of considering intervening time for system adjustments in terms of total cost and system robustness.**

***Index Terms*—Benders decomposition, branch-and-cut algorithms, contingency constraints, non-simultaneous failures, unit commitment.**

NOMENCLATURE

Index Sets and Indices

1) $\mathcal{N}$: the set of buses, indexed by i, j
2) $\mathcal{G}$: the set of generators, indexed by g, $G = |\mathcal{G}|$
   - $\mathcal{G}_i$: the set of generators located at bus i
3) $\mathcal{E}$: the set of transmission lines, $E = |\mathcal{E}|$
   - $\mathcal{E}_{.i}$: the set of lines oriented into bus i
   - $\mathcal{E}_{i.}$: the set of lines oriented out of bus i
   - (i, j): the head bus i and tail bus j of line e
4) $\mathcal{C}$: the set of all N-1-1 contingencies, $C = |\mathcal{C}|$
   - c : contingency index, $c \in \{1, \ldots, C\}$
   - $\boldsymbol{c}$: a binary vector that prescribes a contingency
5) $\mathcal{T}$: the set of time periods, indexed by t, $T = |\mathcal{T}|$

Parameters

1) B_e: susceptance of line e
2) $\overline{\boldsymbol{f}}$: vector of line capacities
3) d_i^t: demand at bus i in period t
4) $\boldsymbol{d}^t$: vector of demands in time period t
5) $\underline{p}_g, \overline{p}_g$: capacity lower/upper bounds for generator g
6) $\underline{r}_g(\boldsymbol{x})$: vector of ramp-down rates given unit commitment vector $\boldsymbol{x}$
7) $\overline{r}_g(\boldsymbol{x})$: vector of ramp-up rates given unit commitment vector $\boldsymbol{x}$
8) $c^s(\boldsymbol{x})$: start-up and shut-down costs given unit commitment vector $\boldsymbol{x}$
9) $c^p(\boldsymbol{p})$: production cost given generation level vector $\boldsymbol{p}$
10) o_e: allowable line overload factor during secondary contingency periods
11) ε: load shedding threshold, $\varepsilon \in [0, 1]$
12) H, A: incidence matrix indicating the location of generators and the start/end of transmission lines

Decision Variables

1) $\boldsymbol{x} \in \{0, 1\}^{G \times T}$: unit commitment vector
2) $\boldsymbol{x}_g \in \{0, 1\}^T$: generator g unit commitment vector
3) $\boldsymbol{p}^t, \boldsymbol{f}^t, \boldsymbol{\theta}^t$: vectors of generation levels, power flows, and phase angles, respectively, in time period t under the no-contingency (base) scenario
4) p_g^t, f_e^t, θ_i^t: the generation level of unit g, the power flow on transmission line e, and the phase angle on bus i, respectively, in time period t under the no-contingency (base) scenario
5) $\boldsymbol{p}^{ct}, \boldsymbol{f}^{ct}, \boldsymbol{\theta}^{ct}, \boldsymbol{q}^{ct}$: vectors of generation levels, power flows, phase angles, and loss-of-load, respectively, in time period t under contingency scenario c.
6) $p_g^{ct}, f_e^{ct}, \theta_i^{ct}, q_i^{ct}$: the generation level of unit g, the power flow on transmission line e, the phase angle on bus i, and the loss-of-load at bus i, respectively, in time period t under contingency scenario c.

Manuscript received April 8, 2016; revised August 12, 2016; accepted September 19, 2016. Date of publication October 4, 2016; date of current version June 16, 2017. The work of Z. Guo was supported in part by the Sustainable Transportation Energy Pathways program and in part by the National Transportation Center on Sustainability, University of California, Davis, CA, USA. The work of R. L.-Y. Chen and J.-P. Watson was supported in part by the Sandia's Laboratory Directed Research and Development program. The work of N. Fan was supported in part by the University of Arizona Renewable Energy Network Faculty Exploratory Grants. Paper no. TPWRS-00555-2016.

Z. Guo is with the Energy Systems Division, Argonne National Laboratory, Lemont, IL 60439 USA (e-mail: zguo@anl.gov).

R. L.-Y. Chen is with the Quantitative Modeling and Analysis Department, Sandia National Laboratories, Livermore, CA 94551 USA (e-mail: rlchen@sandia.gov).

N. Fan is with the Department of Systems and Industrial Engineering, University of Arizona, Tucson, AZ 85721 USA (e-mail: nfan@email.arizona.edu).

J.-P. Watson is with the Discrete Math and Optimization Department, Sandia National Laboratories, Albuquerque, NM 87185 USA (e-mail: jwatson@sandia.gov).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TPWRS.2016.2612680

I. INTRODUCTION

THE North American Electric Reliability Corporation (NERC) develops and enforces standards to ensure the reliability of power systems in North America. The NERC Transmission Planning Standard TPL-001-1 [1] defines system performance requirements under both normal and various contingency conditions. Among contingency conditions, the loss of a single system component (N-1) and the near simultaneous loss of multiple system components (N-k) are well studied.

However, a contingency criterion considering *non-simultaneous* failures of two components has not attracted much attention until recently (see [3], [8]). This contingency criterion, known as N-1-1, refers to the consecutive loss of two components with an intervening time period for operator adjustments. Because the probability of near simultaneous failures of multiple system components is comparably low, considering intervening time for operator adjustments may yield less conservative and more economical unit commitment decisions. In [3], the authors performed an N-1-1 contingency power-flow analysis of the US Midwest ISO's balancing area. More recently, [8] used interdiction methods to study N-1-1 contingency constrained optimal power flows with fixed unit commitment decisions.

As in [8], we assume that an N-1-1 contingency scenario specifies the loss of a generating unit or a transmission line, followed by system adjustments (e.g., generator re-dispatch). The system then experiences the loss of an additional generator or line at a later time period. Thus, there are three distinct time periods in a given N-1-1 contingency scenario. The *Base Case* refers to the time periods in which "the power system is in normal steady-state operation, with all components in service that are expected to be in service" [35]. The first loss of a component is referred to as the *Primary Contingency*, while the second loss is referred to as the *Secondary Contingency*.

Here, we study the unit commitment (UC) problem under N-1-1 contingency constraints. UC involves determination of a minimal-cost "on-off" schedule of generating units and their respective dispatch levels, subject to physical and operational constraints, in order to satisfy forecasted demand in each time period of the subsequent operating day. The basic UC problem, discounting contingencies, is well-studied; relevant efforts were reviewed by [12] and more recently by [14], [27].

The UC problem with contingency constraints has received increasing attention from academics and practitioners following the 2003 Northeast blackout in North America. The primary objective in enforcing contingency constraints is to ensure that power system operations are sufficiently robust to sudden losses of system components. Typical contingency constraints ensure that the system can continue to perform "as is" under a single component failure, e.g., N-1. Under more stringent multiple-failure contingency constraints, e.g., N-k, the system must continue to perform at an acceptable level, with some amount of load shedding and system overload permitted during the recovery period.

References [11], [15] consider N-1 contingency constrained UC, and employ line switching to alleviate congestion and yield a more economical dispatch of generation resources. References [17], [18] use robust optimization to determine an optimal dispatch schedule under the worst-case N-k contingency scenario. Finally, [4], [5] introduce a new N-k-ε criterion which dictates that at least $(1-\varepsilon^j)$ fraction of the total system demand be met following the failures of j (for $j = 1, \ldots, k$) system components; several decomposition methods were proposed to solve the resulting large-scale optimization models.

More recently, [13] develops a comprehensive robust UC model subject to N-k contingency constraints, interval uncertainties associated with load and wind, and quick-start units. Reference [28] describes a probabilistic framework for failure-constrained UC with uncertain renewable energy sources where failure probabilities for fixed sets of generator and transmission outage scenarios are modeled as explicit functions of the binary scheduling variables and penalties on expected loss of load are imposed. Reference [29] presents an approach for joint energy and reserve scheduling for UC with reliability constraints for the day-ahead market, where demand must be met with a specified probability under any simultaneous loss of generating units. An α-quantile measure is used to specify the confidence level of meeting demand under N-k failures and uncertain wind power. Finally, [31] proposed a two-stage robust optimization framework for joint energy and reserve scheduling under N-k contingency constraints. A Benders decomposition solution framework based on a tighter formulation and strong valid inequalities is presented.

In contrast to the above works, we consider N-1-1 contingencies, as defined by NERC standard TPL-001-1 [1]. During the base (no-contingency) case, and during time periods after the primary loss, all thermal limits must be within applicable ratings and loss-of-load is not permitted as a recourse action. During time periods after the secondary loss, controlled load shedding and overloads of transmission lines are allowed for emergency control. The key difference between our model and existing work is the consideration of intervening time between the primary and secondary contingencies, which requires a coupling of contingencies across time periods. This extension to consider the timing of failures results in a combinatorial explosion in the number of possible contingencies (compared to N-2) and significant modeling challenges associated with explicitly defining the three distinct time periods. While we do not explicitly model uncertainties associated with load or renewable resources, such extensions under a robust optimization framework can be incorporated with appropriate enhancements to the separation oracle defined in Section III-B. Finally, to solve the UC problem with N-1-1 contingency constraints, we introduce a novel decomposition method combining branch-and-cut and a temporal decomposition of the associated separation oracle.

The remainder of this paper is organized as follows. In Section II, we briefly introduce the baseline UC problem and study the impact of imposing N-1-1 contingency constraints. We then formulate the N-1-1 contingency-constrained UC problem as a large-scale mixed-integer linear program (MILP). In Section III, we describe our novel solution strategy for this new model. Numerical experiments on several IEEE test systems are considered in Section IV, where we perform a detailed analysis comparing the impacts of different contingency criteria on UC solutions. Finally, we conclude in Section V with a summary of our results and contributions.

II. Unit Commitment Models

We begin by introducing the baseline unit commitment (BUC) problem. The objective of the BUC problem is to determine a minimal-cost on/off schedule and corresponding dispatch levels for a set of thermal generating units under the no-contingency scenario. Building on the BUC problem, we then introduce constraints to enforce the N-1-1 contingency criterion. We refer to the extended problem as the N-1-1 CCUC problem, or for conciseness simply N-1-1.

A. The Baseline Unit Commitment Problem

The BUC problem described below is based on the deterministic UC formulations introduced in [21] and [22]. We extend these formulations to include a DC approximation of power flow on the transmission network. The BUC problem is formulated as follows:

$$\min_{\boldsymbol{x},\boldsymbol{f},\boldsymbol{p},\boldsymbol{\theta}} \quad \boldsymbol{c}^s(\boldsymbol{x}) + c^p(\boldsymbol{p}) \tag{1a}$$

$$\text{s.t.} \quad \boldsymbol{x} \in \mathcal{X} \tag{1b}$$

$$H\boldsymbol{p}^t + A\boldsymbol{f}^t = \boldsymbol{d}^t,\ \forall t \tag{1c}$$

$$B_e(\theta_i^t - \theta_j^t) - f_e^t = 0,\ \forall e = (i,j), t \tag{1d}$$

$$|\boldsymbol{f}^t| \le \overline{\boldsymbol{f}},\ \forall t \tag{1e}$$

$$\underline{p}_g x_g^t \le p_g^t \le \overline{p}_g x_g^t,\ \forall g, t \tag{1f}$$

$$\underline{r}_g(\boldsymbol{x}_g) \le p_g^t - p_g^{t-1} \le \overline{r}_g(\boldsymbol{x}_g),\ \forall g, t \tag{1g}$$

The objective (1a) is to minimize the sum of startup and shutdown cost $\boldsymbol{c}^s(\boldsymbol{x})$ and generation cost $c^p(\boldsymbol{p})$. With dispatch levels prescribed by $\boldsymbol{p}$, $c^p(\boldsymbol{p})$ is often specified using a convex quadratic function for thermal generation units. Constraints (1b) enforce generator minimum uptime/downtime requirements and compute startup and shutdown cost as a function of the units committed. The full description of these constraints is provided in Appendix A; here, $\mathcal{X}$ abstractly denotes the corresponding feasible set. Constraints (1c)-(1g) implement economic dispatch under a DC power flow model, for all time periods. They include, in order: power balance at each bus; power flows on lines; capacity limits for transmission lines; generator dispatch lower and upper bounds; and generator ramping limits across two consecutive time periods. By employing piecewise linearization of the quadratic cost function $c^p(\boldsymbol{p})$, the BUC problem (1) can be reformulated as a MILP.

Remark 1: We do not explicitly impose reserve margins in the BUC as N-1-1 compliancy is a stronger reliability requirement; not only does it ensure sufficient generation reserves for all contingency scenarios but additionally considers the placement of these reserves given constraints on transmission availability and capacity (see also [11]).

B. N-1-1 Contingency Constrained Operations

1) Set of All N-1-1 Contingencies: An N-1-1 contingency refers to the loss of two system components in different time periods. We assume losses are possible for any generating unit(s) and/or transmission line(s). Considering all pairs of time periods for primary and secondary losses, and the possible loss of any generating unit and / or transmission line, the set $\mathcal{C}$ of all N-1-1 contingency scenarios is defined as follows:

$$\mathcal{C} = \Bigg\{ \boldsymbol{c} \in \{0,1\}^{(G+E)\times T} : \tag{2a}$$

$$\sum_{g\in\mathcal{G}} c_g^t + \sum_{e\in\mathcal{E}} c_e^t \le 1, \forall t \in \mathcal{T} \tag{2b}$$

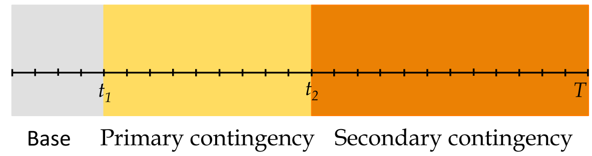

Fig. 1. Three non-overlapping periods of an N-1-1 contingency scenario.

$$\sum_{t\in\mathcal{T}} c_g^t \le 1, \quad \forall g \in \mathcal{G} \tag{2c}$$

$$\sum_{t\in\mathcal{T}} c_e^t \le 1, \quad \forall e \in \mathcal{E} \tag{2d}$$

$$\sum_{t\in\mathcal{T}}\sum_{g\in\mathcal{G}} c_g^t + \sum_{t\in\mathcal{T}}\sum_{e\in\mathcal{E}} c_e^t = 2. \Bigg\} \tag{2e}$$

An N-1-1 contingency scenario $\boldsymbol{c} \in \mathcal{C}$ specifies the following: (i) two time periods, denoted by t_1 and t_2, respectively representing the time periods of primary loss and secondary losses and (ii) one failed component in each of the two periods, denoted by $c_g^{t_1} = 1$ or $c_e^{t_1} = 1$ and $c_g^{t_2} = 1$ or $c_e^{t_2} = 1$. In the definition of set $\mathcal{C}$ given by (2), we do not explicitly reference the two failed periods t_1 and t_2. However, since each contingency scenario defined by constraint set (2) specifies two distinct component failures in two distinct time periods (2e), t_1 and t_2 are implicitly defined by the two distinct failure time periods, with the first failure time corresponding to t_1 and the second failure time corresponding to t_2. Constraints (2b) dictate that at most one component can fail in any given time period. Constraints (2c) and (2d) require that each component fails at most once. Constraint (2e) requires that exactly two distinct components fail. Based on (2), there are $\binom{T}{2} = \frac{T(T-1)}{2}$ possible pairs of time periods for primary and secondary losses, $G + E$ possible primary losses, and $G + E - 1$ possible secondary losses. Thus, the set $\mathcal{C}$ has cardinality $|\mathcal{C}| = C = \frac{T(T-1)}{2}(G + E)(G + E - 1)$. Clearly, even for moderate-sized power systems and small numbers of time periods, solution of the N-1-1 model will pose a considerable computational challenge.

2) N-1-1 contingency requirements: As illustrated in Fig. 1, an N-1-1 contingency scenario is composed of three non-overlapping periods, defined as follows:

1) *Base* $(t \in \{1,\ldots,t_1 - 1\})$. The system operates under normal conditions with no failed components. Non-anticipativity is enforced during this state, such that the operating state (e.g., generation outputs and power flows) is fixed regardless of the specific impeding contingency scenario.
2) *Primary contingency* $(t \in \{t_1,\ldots,t_2 - 1\})$. The system operates under a single component failure. At time period t_1, the system observes the primary loss and transitions from the nominal operating state (prescribed by $\boldsymbol{p}, \boldsymbol{f}, \boldsymbol{\theta}$) to the contingency c operating state (prescribed by $\boldsymbol{p}^c, \boldsymbol{f}^c, \boldsymbol{\theta}^c$). If the outaged component is a generator g, $p_g^{t_1} = 0$. For all other generators $g' \in \mathcal{G} \setminus g$, $p_{g'}^{t_1}$ is ramp-constrained by the generator's dispatch level in period $t_1 - 1$.

3) *Secondary contingency* ($t \in \{t_2, \ldots, T\}$). The system operates with two failed components. Per NERC reliability standards, controlled load shedding and line overloads are permissible. Therefore, the line overload factor o_e and allowable load shedding ε (given as a fraction of total demand) can be utilized to alleviate operational infeasibilities.

For conciseness, we introduce in the following "in the contingency" $\boldsymbol{w}$ indicators, and allowable loss-of-load $\boldsymbol{h}$ and line overload $\boldsymbol{o}$ quantities:

$$w_e^{ct} = \sum_{\tau=1}^{t} c_e^{c\tau},\ \forall e, t \text{ and } w_g^{ct} = \sum_{\tau=1}^{t} c_g^{c\tau},\ \forall g, t \tag{3}$$

$$h_i^{ct} = \begin{cases} 0, & \forall i, t = 1, \cdots, t_2^c - 1 \\ d_i^t, & \forall i, t = t_2^c, \cdots, T \end{cases} \tag{4}$$

$$o_e^{ct} = \begin{cases} 0, & \forall e, t = 1, \cdots, t_2^c - 1 \\ o_e, & \forall e, t = t_2^c, \cdots, T \end{cases} \tag{5}$$

First, observe that each contigency scenario prescribes Constraints (4) and (5) require that during secondary contingency periods, the allowable loss-of-load and overload factor are equal to d_i^t and o_e, respectively. These values are zero in all other periods.

Then, $\forall \boldsymbol{c} \in \mathcal{C}$, the contingency dispatch operation in periods $t \in \{t_1^c, \ldots, T\}$ is constrained as follows:

$$H\boldsymbol{p}^{ct} + A\boldsymbol{f}^{ct} + \boldsymbol{q}^{ct} = \boldsymbol{d}^t,\ \forall t \tag{6a}$$

$$B_e(\theta_i^{ct} - \theta_j^{ct})(1 - w_e^{ct}) - f_e^{ct} = 0,\ \forall e = (i,j), t \tag{6b}$$

$$|f_e^{ct}| \le \overline{f}_e(1 - w_e^{ct})(1 + o_e^{ct}),\ \forall e, t \tag{6c}$$

$$\underline{p}_g x_g^t(1 - w_g^{ct}) \le p_g^{ct} \le \overline{p}_g x_g^t(1 - w_g^{ct}),\ \forall g, t \tag{6d}$$

$$\underline{r}_g(\boldsymbol{x}_g)(1 - w_g^{ct}) \le p_g^{ct} - p_g^{c,t-1} \le \overline{r}_g(\boldsymbol{x}_g)(1 - w_g^{ct}),\ \forall g, t \tag{6e}$$

$$\boldsymbol{q}^{ct} \le \boldsymbol{h}^{ct},\ \forall t \tag{6f}$$

$$\mathbf{1}^\top \boldsymbol{q}^{ct} \le \varepsilon \mathbf{1}^\top \boldsymbol{h}^{ct},\ \forall t \tag{6g}$$

$$p_g^{ct_1^c - 1} - p_g^{t_1^c - 1} = 0,\ \forall g \in \{g | c_g^{t_1^c} = 0\} \tag{6h}$$

Contingency constraints include, in order of appearance: power balance at each bus, with unsatisfied demand $\boldsymbol{q}^{ct}$ (6a); power flows on each line (6b); line capacity bounds (6c) with overload factor o_e during secondary contingency periods; generation dispatch bounds (6d); generator ramping limits (6e); upper bound on loss-of-load at each bus (6f); threshold for total loss-of-load in periods after secondary loss (6g); and the non-anticipativity constraint for generators not in the contingency in period t_1^c (6h). Observe that the secondary failure time period t_2^c does not explicitly appear in constraint set (6), but t_2^c is implied by intermediate variables $\boldsymbol{h}^{ct}$ and $\boldsymbol{o}^{ct}$ defined by (4) and (5), where the values of these parameters explicitly depend on t_2^c.

C. Full Formulation

The optimization objective in the N-1-1 model is to find a minimum-cost UC under the no-contingency scenario such that a feasible recourse power flow exists under each N-1-1 contingency scenario. The full formulation is obtained by combining the BUC model (1) with the full set of contingency constraints (6), one for each contingency scenario $\boldsymbol{c} \in \mathcal{C}$. The full formulation of the N-1-1 CCUC model is then given as follows:

$$\min_{\substack{\boldsymbol{x},\boldsymbol{f},\boldsymbol{p},\boldsymbol{\theta} \\ \boldsymbol{f}^c,\boldsymbol{p}^c,\boldsymbol{q}^c,\boldsymbol{\theta}^c}} \quad c^s(\boldsymbol{x}) + c^p(\boldsymbol{p}) \tag{7a}$$

$$\text{s.t.} \quad \text{Constraints (1b)-(1g)} \tag{7b}$$

$$\text{Constraints(6a)-(6h)},\ \forall \boldsymbol{c} \in \mathcal{C} \tag{7c}$$

Model (7) is an extremely large-scale MILP due to the full set of constraints (7c), one for each contingency scenario. The objective (7a) includes only the unit commitment cost and the no-contingency scenario generation cost. However, extension to consider worst-case cost is straightforward. Following established models ([11], [4], [5]), we ignore costs during a contingency state, as the primary concern of system operators during a contingency is to ensure operational feasibility and system stability.

Remark 2: In the N-1-1 model, there are $\frac{T(T-1)}{2}(G + E)(G + E - 1)$ sets of constraints (7c), which collectively ensure that a feasible recourse power flow exists in each contingency scenario. When defining the full set of contingency scenarios (2), we assumed that when a primary contingency component fails, its failure persists for the remainder of the planning horizon. We can relax this assumption through introduction of a new integer parameter $\tau \ge 1$ that prescribes the number of time periods until the primary contingency component is returned to service. Under this assumption we do not need to consider all time period pairs for the primary and secondary contingency. Rather, it then suffices to consider all pairs of time periods whose difference is less than τ. Then, there are $(T-1) + (T-2) + \cdots + (T-\tau) = \frac{(2T-\tau-1)\tau}{2}$ pairs of primary and secondary failure periods t_1 and t_2.

III. Solution Approaches

The full MILP model (7) can be solved using an iterative algorithm like Benders decomposition (BD); direct solution via the extensive form is not practical, as we show subsequently in Section IV. In applying BD, we first decompose the full problem into a master problem (MP), defined by (7a)-(7b), and a set of subproblems (SP), defined by (7c), one for each contingency $\boldsymbol{c} \in \mathcal{C}$. The MP prescribes the unit commitment vector $\boldsymbol{x}$ and the no-contingency economic dispatch $(\boldsymbol{f}, \boldsymbol{p}, \boldsymbol{\theta})$. The subproblems $\text{SP}(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{c})$ are based on Constraint (6), with the following augmentations:

1) Addition of variables $\boldsymbol{s}$ to indicate the amount of load shedding above the allowable threshold ε. This ensures relatively complete recourse.
2) Addition of the trivial objective function $\min\ \mathbf{1}^\top \boldsymbol{s}$.

The objective of the SP sub-problem is to minimize the amount of demand shed above the allowable threshold. If the objective value of SP$(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{c})$, given by z, is zero then the current solution $\boldsymbol{x}, \boldsymbol{p}$ can survive contingency scenario $\boldsymbol{c}$. Otherwise, a Benders feasibility cut can be added to the MP to eliminate the infeasible solution $(\boldsymbol{x}, \boldsymbol{p})$.

In state-of-the-art algorithms for contingency-constrained UC problems, BD is typically implemented as a cutting plane algorithm (CPA) ([4], [5], [22], [17], [19], and [20]). Although easy to implement, CPAs have limited computational tractability because at each iteration of the algorithm an integer program MP must be solved to select a candidate UC decision. We next outline a branch-and-cut algorithm (BCA) that avoids this drawback by only exploring the branch-and-bound tree corresponding to the UC $\boldsymbol{x}$ once. BCA is a branch-and-bound algorithm in which cutting planes are generated within the branch-and-bound tree.

A. Branch-and-Cut Algorithm

Recent advances in optimization solver technology, both in commercial (IBM CPLEX [7] and Gurobi [10]) and non-commercial (SCIP [16] and DIPS [6]) packages, have significantly simplified the implementation of branch-and-cut algorithms by enabling the addition of valid inequalities directly within the branch-and-bound (B&B) tree, thus avoiding the need to repeatedly explore the branch-and-bound tree defined by the binary UC variables $\boldsymbol{x}$. In IBM CPLEX, branch-and-cut algorithms can be implemented using IloCplex.Callback functions (e.g., IloCplex.LazyConstraintCallback). In recent years, work on a number of combinatorial problems (e.g., survivable network design [9] and the minimum tool booth [2]) have shown that a significant reduction, often an order of magnitude or better, in computational time can be achieved using a BCA compared to a CPA, with increasing reductions in runtime for harder and larger combinatorial problems. However, despite the computational advantages of BCA over CPA very few works, with the exception of [30] which proposed a BCA for solving a two-stage chance-constrained UC problem, have employed a branch-and-cut framework for UC optimization. This motivates our development of a hybrid branch-and-cut algorithm for N-1-1.

Let the linear programming relaxation of MP, the *relaxed master problem* (RMP), be given as follows:

$$\min_{\boldsymbol{x},\boldsymbol{f},\boldsymbol{p},\boldsymbol{\theta}} \quad c^s(\boldsymbol{x}) + c^p(\boldsymbol{p}) \tag{8a}$$

$$\text{s.t.} \quad \boldsymbol{x} \in \mathcal{X} \tag{8b}$$

$$\boldsymbol{0} \le \boldsymbol{x} \le \boldsymbol{1} \tag{8c}$$

$$\text{(1c)-(1g).} \tag{8d}$$

Let L be the list of nodes of the B&B tree that remain to be explored. Initially, L contains only the root node o with no branching constraints. Before initializing the BCA, a number of valid inequalities are added to the initial RMP to strengthen the root node LP relaxation. We refer to these valid inequalities as the cut pool P. We note that in BCA implementations it is important to strengthen the root node adequately to avoid unnecessary exploration of the B&B tree. Starting with an empty P results in very slow pruning because many infeasible nodes will only be pruned late in the search. However, adding too many valid inequalities to P slows down the LP relaxation solves at each node. Thus, there is a significant trade-off between strengthening the root node LP relaxation, and thus avoiding unnecessary exploration of the B&B tree, and overburdening the RMP, and thus increasing per-node LP solve times. This trade-off is more art than science and may be very specific to the application domain. In our BCA implementation, we initialize P with the valid inequalities introduced subsequently.

Let λ_1 and λ_2 be the capacity of the smallest and the second smallest generators, respectively. Then the following are valid inequalities:

$$\sum_{g \in G} x_g^t \ge 3, \quad \forall t = 2, \cdots, T \tag{9a}$$

$$\sum_{g \in G} \overline{p}_g x_g^1 - \lambda_1 \ge \boldsymbol{1}^\top \boldsymbol{d}^1 \tag{9b}$$

$$\sum_{g \in G} \overline{p}_g x_g^t - (\lambda_1 + \lambda_2) \ge \boldsymbol{1}^\top \boldsymbol{d}^t, \quad \forall t = 2, \cdots, T \tag{9c}$$

Constraints (9a) require that at least three generators be on at any given time period after period one, since two generators may fail in consecutive periods. Constraint (9b) dictates that in period 1 the maximum capacity across all committed generators minus the capacity of smallest generator in the system must be greater than the total demand of period 1. Constraints (9c) require the total capacity across all committed generators minus the aggregate capacity of the two smallest generators must be larger than the demand, for each period after the first period. We have considered only a simple set of valid inequalities for P and significant research efforts will be required to identify stronger valid inequalities to initialize P. For example, the *generation outage constraints* proposed by [31] and the *combinatorial inequalities* proposed by [32], both corresponding to the dual of simplified adversarial bilevel program, have the potential to significantly strengthen the master problem relaxation and improve the convergence of BCA.

Solving the RMP for a node o means solving the RMP with associated branching constrained defined by o. Node o prescribes the subset of discrete variables that are fixed at that particular node of the B&B tree. In a depth-first variant of the BCA, branching is performed until an integer solution is identified. Only when an integer solution is identified are violated inequalities screened for each contingency scenario. At the opposite extreme, violated inequalities for each contingency scenario may be screened after each node solve, resulting in the breath-first variant of the BCA. The former, depth first variant, may result in a large node list L and the later may result in the addition of a large number of violated inequalities. A compromise between these two extremes may be achieved by defining a branching depth parameter to control the trade-off between branching and cut generation.

B. Temporally Decomposed Bilevel Separation Oracle

Typically, the existence of a feasible power flow for each contingency scenario must be verified explicitly. Such explicit enumeration, however, cannot scale as the number of contingencies

C is extremely large, even for moderate-sized power systems and/or planning horizons. As we demonstrate in our computational experiments, securing the system against a small number of contingency scenarios is empirically sufficient to ensure feasibility against the full set of contingency scenarios $\mathcal{C}$. So, instead of explicitly checking feasibility across all contingency scenarios, we describe a bilevel separation problem to screen a small number of worst-case contingency scenarios that in aggregate "covers" $\mathcal{C}$.

As noted in [4] and [5], significant computational challenges exist in solving bilevel programs for power system vulnerability analysis. In N-1-1 UC, these challenges are further compounded by the fact that the vulnerability analysis problem spans the T planning periods, resulting in a very large-scale bilevel program with $T \times (G + E)$ upper-level binary decision variables. To overcome this computational challenge, we perform a temporal decomposition in which component failures are restricted to a preselected time period pair. We then iterate over all possible time periods pairs, solving a simplified and smaller bilevel separation oracle for each time period pair.

Given a unit commitment schedule $\boldsymbol{x}$, a no-contingency scenario power flow $\tilde{\boldsymbol{p}}$, and a pair of periods t_1 and t_2 denoting the times of the primary and the secondary contingency, respectively, a *Power System Inhibition Problem* (PSIP) can be used to determine the worst-case loss-of-load under any two non-simultaneous component failures. In this bilevel program, the upper-level decision vector $\boldsymbol{c}$ identifies the primary and secondary component losses, and the lower-level decision vectors $(\boldsymbol{f}, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{s}, \boldsymbol{\theta})$ correspond to recourse power flow under the contingency scenario prescribed by $\boldsymbol{c}$.

The set of valid contingency scenarios given a pair of time periods (t_1, t_2) is given as follows:

$$\mathcal{C}(t_1, t_2) = \Bigg\{ \boldsymbol{c} \in \{0,1\}^{(G+E)\times T} : \tag{10a}$$

$$\sum_{g\in G} c_g^t + \sum_{e\in\mathcal{E}} c_e^t = \begin{cases} 1, & \forall t \in \{t_1, t_2\} \\ 0, & \text{otherwise,} \end{cases} \tag{10b}$$

$$c_g^{t_1} + c_g^{t_2} \le 1, \quad \forall g \in \mathcal{G}, \tag{10c}$$

$$c_e^{t_1} + c_e^{t_2} \le 1, \quad \forall e \in \mathcal{E} \Bigg\} \tag{10d}$$

Constraint (10b) dictates that a single component failure occurs in each of time periods t_1 and t_2. Constraints (10c) and (10d) require that each component can fail at most once. We note that the full set of N-1-1 contingency scenarios can alternatively be defined as the union of sets $\mathcal{C}(t_1, t_2)$ for all distinct time period pairs, as follows:

$$\mathcal{C} = \bigcup_{\forall(t_1,t_2)|t_1<t_2} \mathcal{C}(t_1, t_2). \tag{11}$$

In preceding formulations, $\boldsymbol{c}$ was an input parameter. However, in the PSIP $\boldsymbol{c}$ is a decision vector prescribing the two failed components. For notational convenience, we again define binary variables $\boldsymbol{w}$ to indicate whether or not a component failed in the current period or a prior period. Then PSIP$(\boldsymbol{x}, \tilde{\boldsymbol{p}}, t_1, t_2)$ is given as follows:

$$\max_{\substack{\boldsymbol{w} \\ \boldsymbol{c}\in\mathcal{C}(t_1,t_2)}} \quad \min_{\boldsymbol{f},\boldsymbol{p},\boldsymbol{q},\boldsymbol{s},\boldsymbol{\theta}} \quad \mathbf{1}^\top \boldsymbol{s} \tag{12a}$$

$$\text{s.t. } (\boldsymbol{\alpha}^t) \;\; H\boldsymbol{p}^t + A\boldsymbol{f}^t + \boldsymbol{q}^t = \boldsymbol{d}^t, \; \forall t \tag{12b}$$

$$(\hat{\beta}_e^t, \check{\beta}_e^t) \;\; B_e(\theta_i^t - \theta_j^t)(1 - w_e^t) - f_e^t = 0, \; \forall e, t \tag{12c}$$

$$(\hat{\eta}_e^t, \check{\eta}_e^t) \;\; |f_e^t| \le \overline{f}_e^t(1 - w_e^t)(1 - o_e^t), \; \forall e, t \tag{12d}$$

$$(\hat{\zeta}_g^t, \check{\zeta}_g^t) \;\; \underline{p}_g x_g^t(1 - w_g^t) \le p_g^t \le \overline{p}_g^t x_g^t(1 - w_g^t), \; \forall g, t \tag{12e}$$

$$(\hat{\gamma}_g^t, \check{\gamma}_g^t) \underline{r}_g(\boldsymbol{x})(1 - w_g^t) \le p_g^t - p_g^{t-1} \le \overline{r}_g(\boldsymbol{x})(1 - w_g^t), \forall g, t \tag{12f}$$

$$(\boldsymbol{\lambda}^t) \;\; \boldsymbol{q}^t \le \boldsymbol{h}^t, \; \forall t \tag{12g}$$

$$(\mu^t) \;\; \mathbf{1}^\top \boldsymbol{q}^t - s^t \le \varepsilon \mathbf{1}^\top \boldsymbol{d}^t, \; \forall t \tag{12h}$$

$$(\kappa_g) \;\; p_g^{t_1-1} - \tilde{p}_g^{t_1-1}(1 - w_g^{t_1}) = 0, \; \forall g \quad (\kappa_g) \tag{12i}$$

$$\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{s} \ge \mathbf{0} \tag{12j}$$

The dual variables of the corresponding lower-level problem are presented on the left hand side of (12) and are used in deriving a single-level reformulation. The optimization objective (12a) is to maximize the minimum amount of load shed above the allowable threshold given by $\boldsymbol{s}$. Constraint (12i) enforces non-anticipativity and ramp constraints for any generator not in the contingency at time $p_g^{t_1}$, based on its no-contingency scenario output $\tilde{p}_g^{t_1-1}$.

Model (12) is a bilevel program that cannot be solved directly by commercial solvers, but it can be reformulated as a mixed integer linear program (MILP) by dualizing the lower-level linear program, combing the result with the upper maximization problem, and linearizing the resulting bilinear terms in the objective (see Appendix B for details). In the following algorithmic description, the solution of PSIP refers to the solution of the linearized MILP counterpart of (12).

Instead of solving C linear programs (6) at each iteration, we can solve $\frac{1}{2}(T)(T-1)$ mixed-integer linear programs (12) for every time period pairs (t_1, t_2). We refer to this variant algorithm as "Hybrid Branch-and-Cut" and present the full algorithm in Algorithm 1.

IV. Numerical Experiments and Analysis

We tested our models and the proposed HBC algorithms on the IEEE 6-bus, 9-bus, 14-bus, 24-bus, 30-bus and 39-bus systems[1] [24] on a laptop running Mac OS X, with 2.3 GHz Intel Core i7 processor and 8GB of memory. The models and algorithms are implemented in C++ using IBM's Concert Technology Library 2.9 and CPLEX 12.6 MILP solver. For each instance, we considered 12 time periods, and load-shedding threshold ε and line overload factor o_e of 0.15. The reason that we only considering 12 time periods, instead of a more common practice (24 time periods) in day ahead scheduling for UC

[1] All test cases can be obtained from http://pserc.cornell.edu/matpower.

Algorithm 1: Hybrid Branch-and-Cut Algorithm (HBC).

Require: A starting cut pool P;
Initialization: $z^* = +\infty, L = \{o\}$; `/* o has no branching constraints */`
while L is nonempty **do**
  Select a node $\hat{o} \in L$;
  $L \leftarrow L \setminus \{\hat{o}\}$; `/* delete node */`
  Solve RMP for node $\hat{o}$. Let $(\boldsymbol{x}, \boldsymbol{f}, \boldsymbol{p}, \boldsymbol{\theta})$ be the optimal solution and z be the optimal cost;
  if $z \le z^*$ **then**
    if $\boldsymbol{x}$ is fractional **then**
      Branch, resulting in nodes o' and o'',
      $L \leftarrow L \cup \{o', o''\}$; `/* add nodes */`
    else
      foreach (t_1, t_2) pair **do**
        Solve PSIP$(\boldsymbol{x}, \boldsymbol{p}, t_1, t_2)$. Let ω be the optimal objective value;
        if $\omega > 0$ **then**
          add $f - cut$ to P;
          $L \leftarrow L \cup \{\hat{o}\}$; `/* put back node */`
        else
          if $z < z^*$ **then**
            Define new upper bound $z^* \leftarrow z$ and update incumbent $\boldsymbol{x}^* \leftarrow \boldsymbol{x}$
          end
        end
      end
    end
  end
end
return $\boldsymbol{x}^*$

problem, is mainly due to computational tractability. But notice that because the number of combinations of distinct time period pairs explodes with increases in τ instead of T, one can specify $T = 24$ but limit τ to a smaller number in order to maintain computational tractability. From a practical perspective, a small τ can be justified to some extent because usually a failed element can be brought back online in a relatively short period of time. This section is mainly for illustrating our model and algorithm for N-1-1 contingency criterion, instead of guiding real world operations. In addition, we note here that our 0.15 overload factor is conservative and a higher overload factor may be employ to yield a less conservative (more economical) and more computationally tractable solution (as overload factor increases the feasible region of contingency subproblem grows). For a technical approach on selecting line overload factors using risk analysis we refer to [34]. The peak load factor for each system was estimated using the Electric Reliability Council of Texas (ERCOT) demand data [26].

We begin with an analysis of optimal commitment and dispatch decisions for the 6-bus system across the different contingency criteria. The single-line diagram of the 6-bus system is shown in Fig. 2, in which green text denotes generator data, with minimum and maximum generation limits specified in parentheses; blue text denotes maximum power flow in the nominal operating state; and red text denotes loads. We added three fast-ramping (and more costly) units $G4$–$G6$ in close proximity to

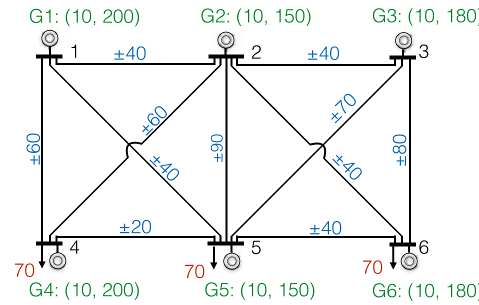

Fig. 2. Single-line diagram of the modified IEEE 6-bus test system.

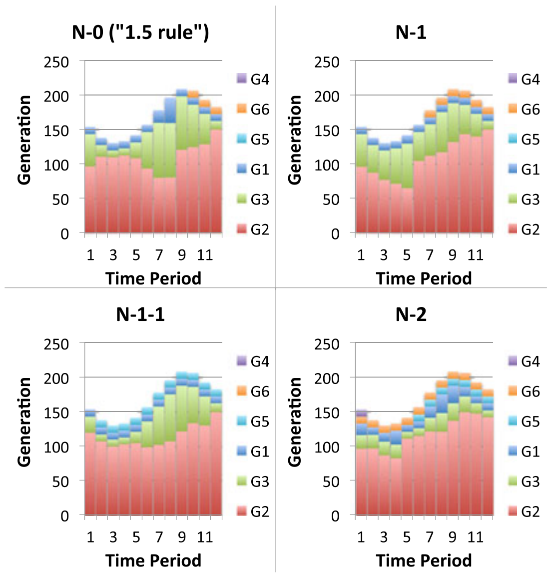

Fig. 3. Optimal dispatch levels for different contingency criteria (6-bus).

TABLE I
UC Decisions Under Different Contingent Criteria for Off-Peak and On-Peak Periods (Modified 6-Bus System)

	Off-peak ($t = 3$)						On-Peak ($t = 9$)					
	G1	G2	G3	G4	G5	G6	G1	G2	G3	G4	G5	G6
$N-0$ ("1.5 rule")	√	√	√				√	√	√			
$N-1$	√	√	√				√	√	√			√
$N-1-1$	√	√	√		√		√	√	√		√	
$N-2$	√	√	√		√	√	√	√	√		√	√

load buses, to ensure system feasibility across the four security criteria: N-0 (with the "1.5 rule" for reserve margins[2]), N-1, N-1-1, and N-2. The specifications of $G4$–$G6$ is included in Appendix C.

The optimal commitment and dispatch decisions for the 6-bus system varies across different contingency criteria, as shown in Fig. 3. For clarity of exposition, we summarize the optimal commitment decisions for two representative time periods (off-peak and on-peak) in Table I, in order to illustrate the differences between UC decisions under different contingency criteria.

[2]The "1.5 rule" is defined as follows: A power system will carry reserves equal to the dispatch level of the largest generator plus one half the dispatch level of the second largest generator. This rule is adopted by several ISOs (e.g., ISO-NE) in the United States.

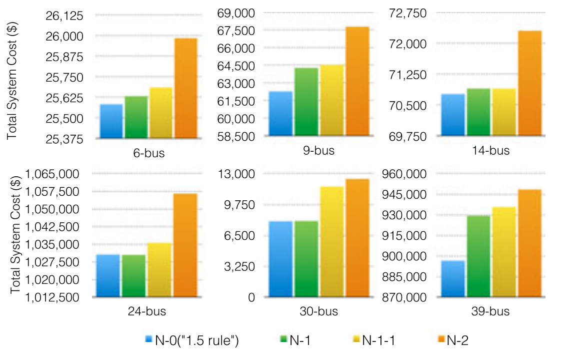

Fig. 4. Total costs across different contingency criteria and test systems.

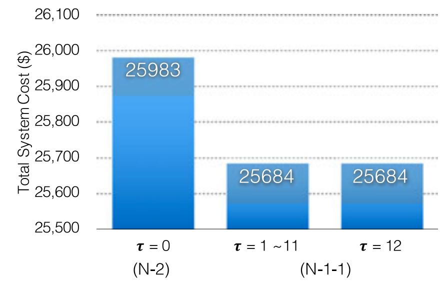

Fig. 5. Total cost as a function of τ for the 6-bus test system.

TABLE II
N-1-1 SOLUTION TIME (SEC.)

Bus	HBC ($\tau = 1$)	HBC ($\tau = 12$)					EF	C&CGA
	Time	B&B nodes	RMP	PSIP		Total time	Time	Time
			Time	Time	Iter #			
6	556	8,065	1,096	348	315	1,444	8,438	479
9	92	1,351	118	351	378	469	3,635	811
14	674	9,908	143	1,517	266	1,660	8,889	*
24	1,052	85,057	1,361	7,259	344	8,620	**	**
30	996	118,062	1,269	7,349	261	8,618	**	**
39	1,173	84,173	853	4,090	1,812	4,943	**	**

Note: *denotes no solution within 24 hours; **denotes out of memory.

Under the "1.5 rule" (N-0), only slower-ramping and lower-cost units $G1$–$G3$ are committed in both time periods. Under the N-1 criterion during the peak period, unit $G6$ is committed in addition to $G1$–$G3$. Under the N-1-1 criterion, we additionally commit $G5$ during the off-peak period and switch from $G6$ to $G5$ during the peak period. The switch from $G6$ to $G5$ reflects a trade-off between production cost and start-up cost. Unit $G6$ has a lower start-up cost but higher production cost than $G5$, see Appendix C. Consequently, when a generator may be needed for a longer time (e.g., under N-1-1), $G5$ is preferable. Lastly, when the contingency criterion includes consideration for two (near) simultaneous failures (N-2) with no intervening time for adjustments, the additional fast-ramping generator $G6$ is committed during the off-peak period. These results are consistent with our intuition that fast-ramping units $G4$–$G6$ are required only if $G1$–$G3$ are not able to meet the security requirements. Additionally, comparing different security criteria, N-2 requires more units to be committed than N-1-1 due to the lack of adjustment time between failures, and more units are committed for N-1-1 than N-1 in order to cover the loss of a second unit in subsequent periods after the primary loss.

Next, we compare total costs under different contingency criteria and test systems, illustrated in Fig. 4. As expected, total cost increases monotonically from N-0 ("1.5 rule"), N-1, and N-1-1 to N-2.[3] In all cases, cost differences under the N-2 and N-1-1 criteria demonstrate the value of intervening time for system adjustments during a multiple-failure contingency scenario. However, the magnitude of these cost differences are system specific. Some test systems (e.g., the 6-bus system) exhibit significant cost differences under the N-1-1 and N-2 criteria but show little cost difference between N-1 and N-1-1. In contrast, the 30-bus system shows the opposite trend: little difference under the N-1-1 and N-2 criterion but significant differences between N-1 and N-1-1.

In the context of *Remark* 2, we assess the impact of varying τ, the maximum time difference between the first and the second component failures, for all six test systems.[4] We note that $\tau = 0$ under N-1-1 corresponds precisely to N-2. Results for the 6-bus system are summarized in Fig. 5. Qualitatively similar results hold for the other test systems. The results indicate that total cost is invariant to changes in τ greater than one. We observe that the computational tractability of the N-1-1 problem is strongly dependent on the parameter τ, see Table II. As τ increases, the number of time period pairs (t_1, t_2) grows exponentially. Our results suggest that solving N-1-1 instances with $\tau = 1$, considering only failures in adjacent time periods, may be a good proxy for the full N-1-1 problem. One of reason for this behavior may be that $\tau = 1$ contains the most "critical" contingency scenarios. If we enforce our UC decisions to be $\tau = 1$ compliant, the other contingencies may be survivable as well. However, rigorously mathematical proof and/or comprehensive numerical experiments on a variety of test systems are required to validate this numerical observation.

Next, we compare the computational performances of HBC (Algorithm 1), the extensive formulation (EF) solved directly with CPLEX, and our implementation of the column-and-cut generation algorithm C&CGA of [33]; the results are reported in Table II. First, we observe that the EF is intractable except for the smaller instances (6-bus to 14-bus) due to the combinatorial explosion in the number of contingency scenarios. While C&CGA performs well for small instances (6-bus and 9-bus), it also fails to solve larger instances in our numerical experiments. This is mainly due to the inefficiencies (RMP bloating) associated with adding full sets of power flow constraints for each identified contingency scenario in the course of branch-and-bound. Except for the (smallest) 6-bus test system, our algorithm outperforms both the EF and C&CGA. Here, our primary objective is to provide evidence of the value of modeling intervening time for operator adjustments in UC contingency criteria. While we have provided some evidence of the computational efficacy of HBC, additional research is required to improve computational performance to permit solution of large-scale instances in operational

[3] We allow the same load shedding and overload criterion when there are two failed elements in the system for both N-1-1 and N-2 cases.

[4] $\tau = 1$ is a special case of the proposed N-1-1 problem and the HBC algorithm can be readily applied to solve the $\tau = 1$ case.

TABLE III
PERCENTAGE OF CONTINGENCY SCENARIOS IDENTIFIED FOR THE N-1-1 CONTINGENCY CRITERION

Bus	G	E	$\tau = 1$			full N-1-1 ($\tau = 12$)		
			M	C	Ratio	M	C	Ratio
6	6	11	5	330	1.52%	10	1,980	0.51%
9	6	9	8	330	2.42%	8	1,980	0.40%
14	10	20	3	990	0.30%	3	5,940	0.05%
24	32	38	3	10912	0.03%	4	65,472	0.01%
30	8	41	10	616	1.62%	53	3,696	1.43%
39	14	46	16	2,002	0.80%	41	12,012	0.34%

time frames. For example, we note that the majority of compute effort is expended solving the separation (PSIP) problem. Thus, heuristic approaches combined with exact approaches may significantly improve overall run times. Additionally, new research efforts to investigate the impact of varying τ, the maximum time between the first and second contingencies, should be conducted. If $\tau = 1$ is a provably effective proxy for the full N-1-1 problem, the number of contingency scenarios under consideration can be significant reduced, which will dramatically reduce run times.

Finally, we discuss the efficacy of implicitly searching for the worst-case contingency scenario by solving PSIPs, rather than explicitly screening the full set of contingency scenarios for each candidate UC solution. We denote the size of the dynamic contingency list in our HBC algorithm by M. The final size of M and the cardinality of the contingency scenario set C are reported in Table III.[5] For each test system, the number of contingency scenarios identified is only a very small fraction of the total possible contingency scenarios, which implies that we only need to generate feasibility cuts using a small set of "sufficient" contingency scenarios to ensure N-1-1 compliancy. This observation is consistent with the analogous observation for general N-k cases, as reported in [4] and [5].

V. CONCLUSION

We have introduced a UC model that explicitly considers non-simultaneous component failures with intervening time by operators for system adjustments. The naive formulation of this model results in an extremely large-scale MILP, due to combinatorial numbers of contingency time period pairs. To overcome this computational challenge, we introduce an efficient HBC algorithm using a temporally decomposed separation oracle. The model and algorithm are tested on multiple IEEE test systems, through which we observe (1) often significant changes in commitment and dispatch decisions for different contingency criteria; (2) the cost benefit of system adjustment time; (3) the general N-1-1 problem can be well-approximated by only considering consecutive failures (i.e., $\tau = 1$); and (4) the number of contingency scenarios identified by our approach represents a very small fraction of the total number.

Allowing for non-simultaneous failures in UC significantly increases the computation burden, relative to other contingency criteria. Although our proposed HBC algorithm is tractable for small and moderate-sized test systems, significant computational challenges remain for larger industrial systems and further algorithmic research in required to achieve scalability. A stronger PSIP formulation or an efficient heuristic will be required to cope with the computational challenge posed by large-scale bilevel programs. Finally, a rigorous mathematic proof and/or a comprehensive computational study is needed to support the idea of considering only one single period between successive failures (i.e., $\tau = 1$) as a proxy for the fully general N-1-1 problem. Aside from computational challenges, understanding the relationship (e.g., the relative magnitude of cost difference) among different contingency criteria is an interesting area for further research.

APPENDIX A
FULL DESCRIPTION OF CONSTRAINTS (1b)

The explicit description of Constraints (1b)[6] is as follows, which include (in order): initial online and offline requirements for generators; minimum uptime in nominal time periods; minimum uptime for the last T_g^u periods; minimum downtime in nominal time periods; minimum downtime for the last T_g^u periods; startup costs; shutdown costs; non-negativity for startup/shutdown costs; and binary constraints for the on/off status of generators.

$$\begin{cases} \sum_{t=1}^{T_g^{u0}} (1 - x_g^t) = 0, \ \forall g \\ \sum_{t=1}^{T_g^{d0}} x_g^t = 0, \ \forall g \\ \sum_{t'=t}^{t+T_g^u-1} x_g^{t'} \geq T_g^u (x_g^t - x_g^{t-1}), \\ \forall g, t = T_g^{u0} + 1, \cdots, T - T_g^u + 1 \\ \sum_{t'=t}^{T} (x_g^{t'} - (x_g^t - x_g^{t-1})) \geq 0, \\ \forall g, t = T - T_g^u + 2, \cdots, T \\ \sum_{t'=t}^{t+T_g^d-1} (1 - x_g^{t'}) \geq T_g^d (x_g^{t-1} - x_g^t), \\ \forall g, t = T_g^{d0} + 1, \cdots, T - T_g^d + 1 \\ \sum_{t'=t}^{T} ((1 - x_g^{t'}) - (x_g^{t-1} - x_g^t)) \geq 0, \\ \forall g, t = T - T_g^d + 2, \cdots, T \\ c_g^{ut} \geq C_g^u (x_g^t - x_g^{t-1}), \ \forall g, t \\ c_g^{dt} \geq C_g^d (x_g^{t-1} - x_g^t), \ \forall g, t \\ c_g^{ut}, c_g^{dt} \geq 0, \ \forall g, t \\ x_g^t \in \{0, 1\}, \ \forall g, t \end{cases} \tag{13}$$

APPENDIX B
SINGLE-LEVEL REFORMULATION OF PSIP

For clarity of exposition, the superscript c identifying contingency scenarios has been removed as the contingency scenario

[5]We only consider generator contingencies in these numerical experiments due to the infeasibility of these small test examples if we include lines failures. So the number of contingency scenarios is calculated as $G(G-1)(T-1)$ for $\tau = 1$ and $G(G-1)T(T-1)/2$ for full N-1-1. The elements in contingency list M are added by iteratively solving PSIPs.

[6]T_g^{u0}/T_g^{d0} denote initial minimal online/offline times; T_g^u/T_g^d denote nominal minimum online/offline times; C_g^u/C_g^d denote startup/shutdown costs.

is not pre-specified, but rather part of the decision making process within the PSIP. In addition, we define T_1 as the set of time periods after t_1, i.e. $T_1 = \{t_1, t_1 + 1, \ldots, T\}$.

$$\max_{\substack{\boldsymbol{c},\boldsymbol{\alpha},\hat{\boldsymbol{\beta}},\check{\boldsymbol{\beta}},\\ \hat{\boldsymbol{\eta}},\check{\boldsymbol{\eta}},\hat{\boldsymbol{\zeta}},\check{\boldsymbol{\zeta}},\boldsymbol{\lambda}\\ \hat{\boldsymbol{\gamma}},\check{\boldsymbol{\gamma}},\boldsymbol{\mu},\boldsymbol{\kappa}}} \sum_{t\in T_1}\sum_{i\in N}(d_i^t\alpha_i^t + h_i^t\lambda_i^t) + \sum_{t\in T_1}\sum_{e\in E} M_e \sum_{t'=t_1}^{t} c_e^{t'}(\hat{\beta}_e^t + \check{\beta}_e^t)$$

$$+ \sum_{t\in T_1}\sum_{e\in E} F_e \left(1 - \sum_{t'=t_1}^{t} c_e^{t'}\right)(\hat{\eta}_e^t + \check{\eta}_e^t)(1 + o_e^{ct})$$

$$+ \sum_{t\in T_1}\sum_{g\in G}(\bar{p}_g\hat{\zeta}_g^t - \underline{p}_g\check{\zeta}_g^t)\left(1 - \sum_{t'=t_1}^{t} c_e^{t'}\right)\tilde{x}_g^t$$

$$\sum_{t\in T_1}\sum_{g\in G}(\bar{r}_g^u(\boldsymbol{x})\hat{\gamma}_g^t + \bar{r}_g^d(\boldsymbol{x})\check{\gamma}_g^t) + \sum_{t\in T_1}\left(\varepsilon\sum_{i\in V} d_i^t\right)\mu^t$$

$$+ \sum_{t\in T_1}\sum_{g\in G} M \sum_{t'=t_1}^{t} c_e^{t'}(\hat{\gamma}_g^t + \check{\gamma}_g^t) + \sum_{g\in G}\kappa_g\tilde{p}_g^{t_1-1}(1 - c_g^{t_1})$$

s.t. Constaints set (10)

$$(f_e^t)\ \alpha_{j_e}^t - \alpha_{i_e}^t + \hat{\eta}_e^t - \check{\eta}_e^t - \hat{\beta}_e^t + \check{\beta}_e^t = 0, \forall e \in E, t \in T_1$$

$$(p_g^t)\ \alpha_{i_g}^t - \delta_g^t + \hat{\zeta}_g^t + \check{\zeta}_g^t + \hat{\gamma}_g^t - \hat{\gamma}_g^{t+1} + \check{\gamma}_g^{t+1} - \check{\gamma}_g^t = 0, \quad \forall g \in G, t \in T_1/\{t_1, T\}$$

$$(p_g^{t_1})\ - \hat{\gamma}_g^{t_1} + \check{\gamma}_g^{t_1} + \kappa_g = 0, \forall g \in G$$

$$(p_g^T)\ \alpha_{i_g}^T - \delta_g^T + \hat{\zeta}_g^T + \check{\zeta}_g^T + \hat{\gamma}_g^T - \check{\gamma}_g^T = 0, \forall g \in G$$

$$(\theta_i^t)\ \sum_{e\in E_{i,\cdot}} B_e(1 + o_e^{ct})(\hat{\beta}_e^t - \check{\beta}_e^t) - \sum_{e\in E_{\cdot,i}} B_e(1 + o_e^{ct})(\hat{\beta}_e^t - \check{\beta}_e^t) = 0, \forall i \in N, t \in T_1$$

$$(q_i^t)\ \alpha_i^t + \lambda_i^t + \mu^t \le 0, \quad \forall i \in N, t \in T_1$$

$$(s^t)\ -\mu^t \le 1, \quad \forall t \in T_1$$

$$\hat{\boldsymbol{\beta}}, \check{\boldsymbol{\beta}}, \hat{\boldsymbol{\eta}}, \check{\boldsymbol{\eta}}, \hat{\boldsymbol{\zeta}}, \check{\boldsymbol{\zeta}}, \boldsymbol{\lambda}, \hat{\boldsymbol{\gamma}}, \check{\boldsymbol{\gamma}}, \boldsymbol{\mu} \le \boldsymbol{0}$$

APPENDIX C
SPECIFICATIONS OF ADDED GENERATORS $G4$–$G6$

TABLE IV
SPECIFICATIONS OF $G4$–$G6$

Specs	$G4$	$G5$	$G6$
Node	3	4	5
Max Output	200	150	180
Min Output	10	10	10
Production Cost	14.415	13.578	13.87
Start Up Cost	55	45	35
Shut Down Cost	5	5	5
Ramp UP	60	40	50
Ramp Down	60	40	50

ACKNOWLEDGMENT

Sandia National Laboratories is a multiprogram laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under Contract DE-AC04-94AL85000.

REFERENCES

[1] *System Performance Under Normal Conditions, North American Electric Reliability Corporation, Standard TPL-001–1*, Feb. 2011. [Online]. Available: http://www.nerc.com/docs/standards/sar/Project_2006-02_TPL-001–1.pdf

[2] L. Bai and P. A. Rubin, "Combinatorial Benders cuts for the minimum toll booth problem," *Oper. Res.* vol. 57, no. 6, pp. 1510–1522, 2009.

[3] D. Chatterjee, J. Webb, Q. Gao, M. Y. Vaiman, M. M. Vaiman, and M. Povolotskiy, "N-1-1 AC contingency analysis as a part of NERC compliance studies at Midwest ISO," in *Proc. 2010 IEEE PES Transm. Distrib. Conf. Expo.*, Apr. 2010, pp. 1–7.

[4] R. L.-Y. Chen, A. Cohn, N. Fan, and A. Pinar, "Contingency-risk informed power system design," *IEEE Trans. Power Syst.* vol. 29, no. 5, pp. 2087–2096, Sep. 2014.

[5] R. L.-Y. Chen, N. Fan, A. Pinar, and J.-P. Watson, "Contingency-constrained unit commitment with post-contingency corrective recourse," *Ann. Oper. Res.*, pp. 1–27, 2014.

[6] *Decomposition for Integer Programing (DIP)*. (2016). [Online]. Available: https://projects.coin-or.org/Dip

[7] *IBM ILOG CPLEX Optimization Studio*. (2016). [Online]. Available: http://www.ibm.com/software/products/en/ibmilogcpleoptistud/

[8] N. Fan, R. Chen, and J.-P. Watson, "N-1-1 contingency-constrained optimal power flow by interdiction methods," in *Proc. 2012 IEEE Power & Energy Soc. General Meeting*, San Diego, CA, USA, Jul. 2012, pp. 1–6.

[9] B. Fortz and M. Poss, "An improved Benders decomposition applied to a multi-layer network design problem," *Oper. Res. Lett.* vol. 37, no. 5, pp. 359–364, 2009.

[10] *Gurobi optimization*. (2016). [Online]. Available: http://www.gurobi.com/

[11] K. W. Hedman, M. C. Ferris, R. P. O'Neill, E. B. Fisher, and S. S. Oren, "Co-optimization of generation UC and transmission switching with N-1 reliability," *IEEE Trans. Power Syst.* vol. 24, no. 2, pp. 1052–1063, May 2010.

[12] B. F. Hobbs, M. H. Rothkopf, R. P. O'Neill, and H.-P. Chao, *The Next Generation of Electric Power UC Models*. Boston, MA, USA: Kluwer, 2001.

[13] B. Hu and L. Wu, "Robust SCUC considering continuous/discrete uncertainties and quick-start units: A two-stage robust optimization with mixed-integer recourse," *IEEE Trans. Power Syst.* vol. 31, no. 2, pp. 1407–1419, Mar. 2016.

[14] N. P. Padhy, "UC—A bibliographical survey," *IEEE Trans. Power Syst.* vol. 19, no. 3, pp. 1196–1205, Aug. 2004.

[15] R. P. O'Neill, K. W. Hedman, E. R. Krall, A. Papavasiliou, and S. S. Oren, "Economic analysis of the N-1 reliable UC and transmission switching problem using duality concepts," *Energy Syst.* vol. 1, no. 2, pp. 165–195, 2010.

[16] *SCIP Optimization Suite*. (2016). [Online]. Available: http://scip.zib.de/

[17] A. Street, F. Oliveira, and J. M. Arroyo, "Contingency-constrained UC with n-K security criterion: A robust optimization approach," *IEEE Trans. Power Syst.* vol. 26, no. 3, pp. 1581–1590, Aug. 2011.

[18] Q. Wang, J.-P. Watson, and Y. Guan, "Two-stage robust optimization for N-k contingency-constrained unit commitment," *IEEE Trans. Power Syst.* vol. 28, no. 3, pp. 2366–2375, Aug. 2013.

[19] Y. Yao, T. Edmunds, D. Papageorgiou, and R. Alvarez, "Trilevel optimization in power network defense," *IEEE Trans. Syst. Man, Cybern., C, Appl. Rev.* vol. 37, no. 4, pp. 712–718, Jul. 2007.

[20] W. Yuan, L. Zhao, and B. Zeng, "Optimal power grid protection through a defender-attacker-defender model," *Rel. Eng. Syst. Safety* vol. 121, pp. 83–89, 2014.

[21] M. Carrión and J. M. Arroyo, "A compuationally efficient mixed-integer linear formulation for the thermal UC problem," *IEEE Trans. Power Syst.* vol. 21, no. 3, pp. 1371–1378, Aug. 2006.

[22] L. Wu and M. Shahidehpour, "Accelerating the Benders decomposition for network-constrained UC problems," *Energy Syst.* vol. 1, no. 3, pp. 339–376, 2010.

[23] *IEEE Reliability Test Data*. (2016). [Online]. Available: http://www.ee.washington.edu/research/pstca/
[24] R. Zimmerman, C. Murillo-Sanchez, and D. Gan, "MATPOWER: A MATLAB power system simulation package 2006," 2009. [Online]. Available: http://pserc.cornell.edu/matpower
[25] J. Aghaei and M.-I. Alizadeh, "Robust N-k contingency constrained unit commitment with ancillary service demand response program," *IET Gener., Transm. Distrib.*, vol. 8, pp. 1928–1936, 2014.
[26] "Actual loads of weather zones report," Elect. Rel. Council Texas, Austin, TX, USA, 2005. [Online]. Available: http://www.ercot.com/content/cdr/html/actual_loads_of_weather_zones
[27] Q. P. Zheng, J. Wang, and A. L. Liu, "Stochastic optimization for unit commitment—A review," *IEEE Trans. Power Syst.* vol. 30, no. 4, pp. 1913–1924, Jul. 2015.
[28] R. Fernández-Blanco, Y. Dvorkin, and M. Ortega-Vazquez, "Probabilistic security-constrained unit commitment with generation and transmission contingencies," *IEEE Trans. Power Syst.*, to be published.
[29] D. Pozo and J. Contreras, "A chance-constrained unit commitment with an n-K security criterion and significant wind generation," *IEEE Trans. Power Syst.* vol. 28, no. 3, pp. 2842–2851, Aug. 2013.
[30] Y. Huang, Q. P. Zheng, and J. Wang, "Two-stage stochastic unit commitment model including non-generation resources with conditional value-at-risk constraints," *Elect. Power Syst. Res.*, vol. 116, pp. 427–438, 2014.
[31] A. Street, A. Moreira, and J. M. Arroyo, "Energy and reserve scheduling under a joint generation and transmission security criterion: An adjustable robust optimization approach," *IEEE Trans. Power Syst.* vol. 29, no. 1, pp. 3–14, Jan. 2014.
[32] R. L.-Y. Chen and C. A. Phillips, "k-edge failure resilient network design," *Electron. Notes Discrete Math.* vol. 41, pp. 375–382, 2013.
[33] L. Zhao and B. Zeng, "Robust unit commitment problem with demand response and wind energy," in *Proc. 2012 IEEE Power & Energy Soc. General Meeting*, 2012, pp. 1–8.
[34] H. Wan, J. D. McCalley, and V. Vittal, "Increasing thermal rating by risk analysis," *IEEE Trans. Power Syst.*, vol. 14, no. 3, pp. 815–828, Aug. 1999.
[35] R. Scott and P. E. Dahman, "N-1–1 contingency analysis using PowerWorld simulator," Power World Corp., Champaign, IL, USA, 2010.

Zhaomiao Guo received the B.S. degree in civil engineering from Tsinghua University, Beijing, China, in 2010, the M.S. degree in transportation engineering from the University of California, Berkeley, CA, USA, in 2011, and both the M.S. degree in agricultural and resources economics and the Ph.D. degree in civil engineering from the University of California, Davis, CA, USA, in 2016. He is currently working as a postdoctoral appointee in the Energy Systems Division, Argonne National Laboratory, Lemont, IL, USA. His research focuses on the reliability and resilience of critical infrastructure systems, with a particular interest in the interactions of transportation and energy systems.

Richard Li-Yang Chen received the B.S. and M.S. degrees in industrial engineering and operations research from the University of California, Berkeley, CA, USA, in 1995 and 1999, respectively, and the Ph.D. degree in industrial and operations engineering from the University of Michigan, Ann Arbor, MI, USA, in 2010. He is a Principal Member of Technical Staff in the Sandia National Laboratories, Livermore, CA, USA. His research focuses on computational stochastic optimization and network design with applications to power systems, telecommunication, and transportation and logistics.

Dr. Chen is a Member of INFORMS.

Neng Fan received the B.S. degree in computational mathematics from Wuhan University, Wuhan, China, in 2004, the M.S. degree in applied mathematics from Nankai University, Tianjin, China, in 2007, and the M.S. and Ph.D. degrees in industrial and systems engineering from the University of Florida, Gainesville, FL, USA, in 2009 and 2011, respectively. He is an Assistant Professor at the University of Arizona, Tucson, AZ, USA. His research focuses on mathematical optimization and data analytics with applications to power systems reliability and renewable energy integration.

Jean-Paul Watson (M'10) received the B.S., M.S., and Ph.D. degrees in computer science. He is a Distinguished Member of Technical Staff in the Discrete Math and Optimization Department, Sandia National Laboratories, Albuquerque, NM, USA. He leads a number of research efforts related to stochastic optimization, ranging from fundamental algorithm research and development, to applications including power grid operations and planning.